







# 11

## Sampling Weights

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### 11.1 Overview

The basic sample design used in TIMSS 1999 was a two-stage stratified cluster design, with schools as the first stage and classrooms as the second. The design required schools to be sampled using a probability proportional to size (PPS) systematic method and classrooms to be sampled with equal probabilities<sup>1</sup>. TIMSS participants adapted the basic design to the requirements of their education systems, with guidance from the TIMSS sampling consultants at Statistics Canada and the sampling referee. Very large countries could add an extra, preliminary stage, where districts or regions were sampled first, and then schools within districts.<sup>2</sup> Also, countries where classes were usually very large could select a subsample of students from sampled classes.<sup>3</sup> Participants used stratification to improve the precision of their samples where appropriate. These adaptations could be quite complex, as may be seen from the information in Appendix D showing how the TIMSS design was implemented in each country.

While the TIMSS multistage stratified cluster design provided very economical and effective data collection in a school environment, it results in differential probabilities of selection for the ultimate sampling elements, the students. Consequently, one student in the assessment does not necessarily represent the same proportion of students in the population as another, as would be the case with a simple random sampling approach. To account for differential probabilities of selection due to the design and to ensure proper survey estimates, TIMSS computed a sampling weight for each participating student. Just as in TIMSS 1995, the ability to provide proper sampling weights was an essential characteristic of an acceptable sample design in TIMSS 1999, since appropriate sampling weights were essential for the computation of accurate survey estimates. This chapter describes the procedures for calculating sampling weights for the TIMSS 1999 data.

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1. The TIMSS sample design is presented in Chapter 2.
2. For example, the United States sampled school districts as primary sampling units and then schools within the school districts.
3. Morocco was the only country to exercise this option in 1999.

## 11.2 Weighting Procedure

The weighting procedure required three steps, reflecting the TIMSS sample design. The first step consisted of calculating a school weight; this also incorporated weighting factors from any additional front-end sampling stages such as districts or regions. A school-level participation adjustment was then made in the school weight to compensate for any sampled schools that did not participate. That adjustment was calculated independently for each explicit stratum.

In the second step a classroom weight was calculated. No classroom-level participation adjustment was necessary, since in most cases a single classroom was sampled in each school. If a school agreed to take part in the study but the sampled classroom refused to participate, the non-participation adjustment was made at the school level. If one of two selected classrooms in a school did not participate, the classroom weight was calculated as though a single classroom had been selected in the first place. The classroom weight was calculated independently for each school.

The third and final step consisted of calculating a student weight. A non-participation adjustment was made to compensate for students that did not take part in the testing. The student weight was calculated independently for each sampled classroom. The basic sampling weight attached to each student record was the product of the three intermediate weights: the first stage (school) weight, the second stage (classroom) weight, and the third stage (student) weight. The overall student sampling weight was the product of these three weights and the two non-participation adjustments, school-level and student-level.

### 11.2.1 The First Stage (School) Weight

The first stage weight represented the inverse of the first stage selection probability assigned to a sampled school. The TIMSS 1999 sample design required that school selection probabilities be proportional to the school size (PPS) school size being enrolment in the target grade. The basic first stage weight for the  $i^{\text{th}}$  sampled school was thus defined as

$$BW_{sc}^i = \frac{M}{n \cdot m_i}$$

where  $n$  was the number of sampled schools,  $m_i$  was the measure of size for the  $i^{\text{th}}$  school, and

$$M = \sum_{i=1}^N m_i$$

where N was the total number of schools in the explicit stratum.

The basic first stage weight also incorporated weighting factors resulting from any additional front-end sampling stages that were applied. The calculation of such weighting factors was similar to that of the first stage weight, since geographical regions were also sampled PPS. The resulting first stage weight in such cases was simply the product of the “region” weight and the first stage weight, as described earlier.

In some countries, schools were selected with equal probabilities. This generally occurred when no reliable measure of school size was available. In some countries also, explicit strata were defined to deal with very large schools or with small schools, and equal probability sampling was necessary in these strata.

Under equal probability sampling, the basic first stage weight for the  $i^{\text{th}}$  sampled school was defined as

$$BW_{sc}^i = \frac{N}{n}$$

where n was the number of sampled schools and N was the total number of schools in the explicit stratum. The basic weight for all sampled schools in an explicit stratum was identical in this context.

### 11.2.2 School Non-Participation Adjustment

First stage weights were calculated for all sampled schools and replacement schools that participated. A school-level participation adjustment was required to compensate for schools that were sampled but did not participate and were not replaced. Sampled schools that were found to be ineligible<sup>4</sup> were removed from the calculation of this adjustment. The school-level participation adjustment was calculated separately for each explicit stratum.

The adjustment was calculated as follows:

$$A_{sc} = \frac{n_s + n_{r1} + n_{r2} + n_{nr}}{n_s + n_{r1} + n_{r2}}$$

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4. A sampled school was ineligible if it was found to contain no eligible (i.e., eighth-grade students). Such schools usually were in the sampling frame by mistake, and included schools that had recently closed, or amalgamated with another school.

where  $n_s$  was the number of originally sampled schools that participated,  $n_{r1}$  and  $n_{r2}$  the number of first and second replacement schools, respectively, that participated, and  $n_{nr}$  the number of schools that did not participate.

The final first stage weight for the  $i^{\text{th}}$  school, corrected for non-participating schools, thus became:

$$FW_{sc}^i = A_{sc} \cdot BW_{sc}^i$$

### 11.2.3 The Second Stage (Classroom) Weight

The second stage weight represented the inverse of the second stage selection probability assigned to a sampled classroom. Although almost all TIMSS 1999 participants sampled intact classrooms using equal probability sampling, it also was permissible to subsample students within classes using PPS techniques. Procedures for calculating sampling weights are presented below for both approaches.

**Equal Probability Weighting:** For the  $i^{\text{th}}$  school, let  $C^i$  be the total number of classrooms and  $c^i$  the number of sampled classrooms. Using equal probability sampling, the final second stage weight assigned to all sampled classrooms in the  $i^{\text{th}}$  school was

$$FW_{cl1}^i = \frac{C^i}{c^i}$$

As a rule,  $c^i$  took the values 1 or 2 and remained fixed for all sampled schools. In those cases where  $c^i$  took the value 2 and only one of the sampled classrooms participated, the second stage weight was adjusted by multiplying it by 2.

**Probability Proportional to Size Weighting:** For the  $i^{\text{th}}$  school, let  $k^{i,j}$  be the size of the  $j^{\text{th}}$  classroom. Using PPS sampling, the final second stage weight assigned to the  $j^{\text{th}}$  sampled classroom in the  $i^{\text{th}}$  school was

$$FW_{cl2}^{i,j} = \frac{K^i}{c^i \cdot k^{i,j}}$$

where  $c^i$  was the number of sampled classrooms in the  $i^{\text{th}}$  school, as defined earlier, and

$$K^i = \sum_{j=1}^{c^i} k^{i,j}$$

Again, usually  $c^j$  took the values 1 or 2 and remained fixed for all sampled schools. In those cases where  $c^j$  took the value 2 and only one of the sampled classrooms participated, the second stage weight was adjusted by multiplying it by 2.

#### 11.2.4 The Third Stage (Student) Weight

The third stage weight represented the inverse of the third stage selection probability attached to a sampled student. Although almost all participants sampled intact classrooms where all eligible students were to be tested, some countries with large classes took a subsample of students from within the sampled classes. Procedures for calculating weights are presented below for both sampling approaches. The third stage weight was calculated independently for each sampled classroom.

**Sampling Intact Classrooms:** If intact classrooms were sampled, then the basic third stage weight for the  $j^{\text{th}}$  classroom in the  $i^{\text{th}}$  school was simply

$$BW_{st1}^{i,j} = 1.0$$

Although in the standard TIMSS data collection each student was assigned one of 8 achievement test booklets<sup>5</sup>, countries were permitted to add a further national booklet as required. Where a country chose to add a national booklet, the basic third stage weight was adjusted to reflect the change in the fraction of students responding to each booklet. The basic third stage weight thus became

$$BW_{st1}^{i,j} = \frac{k_{TIMSS\ 1999}^{i,j} + k_{natl}^{i,j}}{k_{TIMSS\ 1999}^{i,j}}$$

where

$k_{TIMSS\ 1999}^{i,j}$  = number of students assigned a TIMSS 1999 booklet in the  $j^{\text{th}}$  classroom of the  $i^{\text{th}}$  school,

$k_{natl}^{i,j}$  = number of students assigned a national booklet in the  $j^{\text{th}}$  classroom of the  $i^{\text{th}}$  school, and

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5. See Chapter 2 for a description of the TIMSS test design.

$$k_{TIMSS\ 1999}^{i,j} + k_{natl}^{i,j} + k_{ex}^{i,j} = k^{i,j}$$

where  $k_{ex}^{i,j}$  was the number of excluded students<sup>6</sup> that were not assigned any booklet. Note that this number could be zero if there were no excluded students in the classroom.

**Subsampling Students:** If subsampling of students occurred within sampled classrooms, then the basic third stage weight for the  $j^{\text{th}}$  classroom of the  $i^{\text{th}}$  school was

$$BW_{st2}^{i,j} = \frac{k^{i,j}}{s^{i,j}}$$

where  $k^{i,j}$  was the size of the  $j^{\text{th}}$  classroom in the  $i^{\text{th}}$  school, as defined earlier, and  $s^{i,j}$  was the number of sampled students per sampled classroom. The latter number usually remained constant for all sampled classrooms.

When a country added a national booklet to the set of TIMSS 1999 booklets, the basic third stage weight was adjusted to reflect this. The basic third stage weight thus became

$$BW_{st2}^{i,j} = \frac{k^{i,j}}{s^{i,j}} \cdot \frac{s_{TIMSS\ 1999}^{i,j} + s_{natl}^{i,j}}{s_{TIMSS\ 1999}^{i,j}}$$

where

$s_{TIMSS\ 1999}^{i,j}$  = number of sub-sampled students assigned a TIMSS 1999 booklet in the  $j^{\text{th}}$  classroom of the  $i^{\text{th}}$  school,

$s_{natl}^{i,j}$  = number of sub-sampled students assigned a national booklet in the  $j^{\text{th}}$  classroom of the  $i^{\text{th}}$  school, and

$$s_{TIMSS\ 1999}^{i,j} + s_{natl}^{i,j} + s_{ex}^{i,j} = s^{i,j}$$

where  $s_{ex}^{i,j}$  was the number of excluded students that were not assigned any type of booklet. Again, this number could be zero if there were no excluded students in the classroom sub-sample.

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6. Criteria for excluding students from the data collection are presented in Chapter 2.



### 11.2.5 Adjustment for Student Non-Participation

The student non-participation adjustment was calculated separately for each participating classroom as follows:

$$A_{st}^{i,j} = \frac{\frac{s^{i,j} + s^{nr}}{rs}}{\frac{s^{i,j}}{rs}}$$

This adjustment is the inverse of the unweighted student participation rate,  $R_{st}$ , computed for the corresponding classroom:

$$A_{st}^{i,j} = \frac{1}{R_{st}^{i,j}}$$

The third and final stage weight for the  $j^{\text{th}}$  classroom in the  $i^{\text{th}}$  school thus became

$$FW_{st1}^{i,j} = A_{st}^{i,j} \cdot BW_{st1}^{i,j}$$

when intact classrooms were sampled, or

$$FW_{st2}^{i,j} = A_{st}^{i,j} \cdot BW_{st2}^{i,j}$$

when sub-sampling of students within sampled classrooms occurred.

### 11.2.6 Overall Sampling Weights

The overall sampling weight was simply the product of the final first stage weight, the final second stage weight, and the final third stage weight. When intact classrooms were tested the overall sampling weight was

$$W^{i,j} = A_{sc}^{i,j} \cdot BW_{sc}^i \cdot FW_{cl1}^{i,j} \cdot A_{st}^{i,j} \cdot BW_{st1}^{i,j}$$

or

$$W^{i,j} = FW_{sc}^i + FW_{cl1}^{i,j} + FW_{st1}^{i,j}$$

When students were subsampled within classrooms, the overall sampling weight was

$$W^{i,j} = A_{sc}^{i,j} \cdot BW_{sc}^i \cdot FW_{cl2}^{i,j} \cdot A_{st}^{i,j} \cdot BW_{st2}^{i,j}$$

or

$$W^{i,j} = FW_{sc}^i + FW_{cl2}^{i,j} + FW_{st2}^{i,j}$$

It is important to note that sampling weights vary by school and classroom, but that students within the same classroom have the same sampling weights.

### 11.3 Participation Rates

Since lack of participation by sampled schools or students can lead to bias in the results, a variety of participation rates were computed to reveal how successful countries had been in securing participation from their sampled schools. To monitor school participation, three school participation rates were computed: using originally sampled schools only; using sampled and first replacement schools; and using sampled and both first and second replacement schools. Student participation rates were also computed, as were overall participation rates.

#### 11.3.1 Unweighted School Participation Rates

The three unweighted school participation rates that were computed were the following:

$R_{unw}^{sc-s}$  = unweighted school participation rate for originally-sampled schools only,

$R_{unw}^{sc-r1}$  = unweighted school participation rate, including sampled and first replacement schools,

$R_{unw}^{sc-r2}$  = unweighted school participation rate, including sampled, first and second replacement schools.

Each unweighted school participation rate was defined as the ratio of the number of participating schools to the number of originally-sampled schools, excluding any ineligible schools. The rates were calculated as follows:

$$R_{unw}^{sc-s} = \frac{n_s}{n_s + n_{r1} + n_{r2} + n_{nr}}$$

$$R_{unw}^{sc-r1} = \frac{n_s + n_{r1}}{n_s + n_{r1} + n_{r2} + n_{nr}}$$

$$R_{unw}^{sc-s} = \frac{n_s + n_{r1} + n_{r2}}{n_s + n_{r1} + n_{r2} + n_{nr}}$$

### 11.3.2 Unweighted Student Participation Rate

The unweighted student participation rate was computed as follows:

$$R_{unw}^{st} = \frac{\sum_{i,j}^s rs}{\sum_{i,j}^s rs + \sum_{i,j}^s nr}$$

### 11.3.3 Unweighted Overall Participation Rates

Three unweighted overall participation rates were computed for each country. They were as follows:

$R_{unw}^{ov-s}$  = unweighted overall participation rate for originally sampled schools only,

$R_{unw}^{ov-r1}$  = unweighted overall participation rate, including sampled and first replacement schools,

$R_{unw}^{ov-r2}$  = unweighted overall participation rate, including sampled, and first and second replacement schools.

For each country, the overall participation rate was defined as the product of the unweighted school participation rate and the unweighted student participation rate. They were calculated as follows:

$$R_{unw}^{ov-s} = R_{unw}^{sc-s} \cdot R_{unw}^{st}$$

$$R_{unw}^{ov-r1} = R_{unw}^{sc-r1} \cdot R_{unw}^{st}$$

$$R_{unw}^{ov-r2} = R_{unw}^{sc-s} \cdot R_{unw}^{st}$$

### 11.3.4 Weighted School Participation Rates

In TIMSS 1995, the weighted school-level participation rates were computed using school sampling frame information. However, TIMSS 1999 used student-level information instead. The alternate method has two advantages:

- Weighted school participation rates can be easily replicated by all data users since all the required data are available from the international database
- These rates more accurately reflect the current size of the target population since they rely on up-to-date within-school sampling information.

The 1995 method relied on school data as reported on the sampling frame, which often were not up to date with regard to current school enrollment. Conceptually, however, both methods are equivalent when assuming an up-to-date sampling frame, and should yield comparable results in practice.

Three weighted school-level participation rates were computed using the alternate method. They were as follows:

$R_{wtd}^{sc-s}$  = weighted school participation rate for originally-sampled schools only,

$R_{wtd}^{sc-r1}$  = weighted school participation rate, including sampled and first replacement schools,

$R_{wtd}^{sc-r2}$  = weighted school participation rate, including sampled, first and second replacement schools.

The weighted school participation rates were calculated as follows:

$$R_{wtd}^{sc-s} = \frac{\sum_{i,j} BW_{sc}^i \cdot FW_{clx}^{i,j} \cdot FW_{stx}^{i,j}}{\sum_{i,j} FW_{sc}^i \cdot FW_{clx}^{i,j} \cdot FW_{stx}^{i,j}}$$

$$R_{wtd}^{sc-r1} = \frac{\sum_{i,j}^{s+r1} BW_{sc}^i \cdot FW_{clx}^{i,j} \cdot FW_{stx}^{i,j}}{\sum_{i,j} FW_{sc}^i \cdot FW_{clx}^{i,j} \cdot FW_{stx}^{i,j}}$$

$$R_{wtd}^{sc-r2} = \frac{\sum_{i,j}^{s+r1+r2} BW_{sc}^i \cdot FW_{clx}^{i,j} \cdot FW_{stx}^{i,j}}{\sum_{i,j}^{s+r1+r2} FW_{sc}^i \cdot FW_{clx}^{i,j} \cdot FW_{stx}^{i,j}}$$

where both the numerator and denominator were summations over all responding students and the appropriate classroom-level and student-level sampling weights were used. Note that the basic school-level weight appears in the numerator, whereas the final school-level weight appears in the denominator.

The denominator remains unchanged in all three equations and is the weighted estimate of the total enrollment in the target population. The numerator, however, changes from one equation to the next. Only students from originally sampled schools were included in the first equation; students from first replacement schools were added in the second equation; and students from first and second replacement schools were added in the third equation.

### 11.3.5 Weighted Student Participation Rates

The weighted student response rate was computed as follows:

$$R_{wtd}^{st} = \frac{\sum_{i,j}^{s+r1+r2} BW_{sc}^i \cdot FW_{clx}^{i,j} \cdot BW_{stx}^{i,j}}{\sum_{i,j}^{s+r1+r2} BW_{sc}^i \cdot FW_{clx}^{i,j} \cdot FW_{stx}^{i,j}}$$

where both the numerator and denominator were summations over all responding students and the appropriate classroom-level and student-level sampling weights were used. Note that the basic student weight appears in the numerator, whereas the final student weight appears in the denominator. Furthermore, the denominator in this formula was the same quantity that appears in the numerator of the weighted school-level participation rate for all participating schools, sampled and replacement.

### 11.3.6 Weighted Overall Participation Rates

Three weighted overall participation rates were computed. They were as follows:

$$R_{wtd}^{ov-s} = \text{weighted overall participation rate for originally-sampled schools only,}$$

$R_{wtd}^{ov-r1}$  = weighted overall participation rate, including sampled and first replacement schools,

$R_{wtd}^{ov-r2}$  = weighted overall participation rate, including sampled, first and second replacement schools.

Each weighted overall participation rate was defined as the product of the appropriate weighted school participation rate and the weighted student participation rate. They were computed as follows:

$$R_{wtd}^{ov-s} = R_{wtd}^{sc-s} \cdot R_{wtd}^{st}$$

$$R_{wtd}^{ov-r1} = R_{wtd}^{sc-r1} \cdot R_{wtd}^{st}$$

$$R_{wtd}^{ov-r2} = R_{wtd}^{sc-r2} \cdot R_{wtd}^{st}$$

## 11.4 Summary

The multi-stage nature of the TIMSS sampling design means that student have been sampled with varying probabilities. Consequently, if statistics computed from the sample data are to accurately reflect population values, the TIMSS sampling weights must be used when analyzing the data.