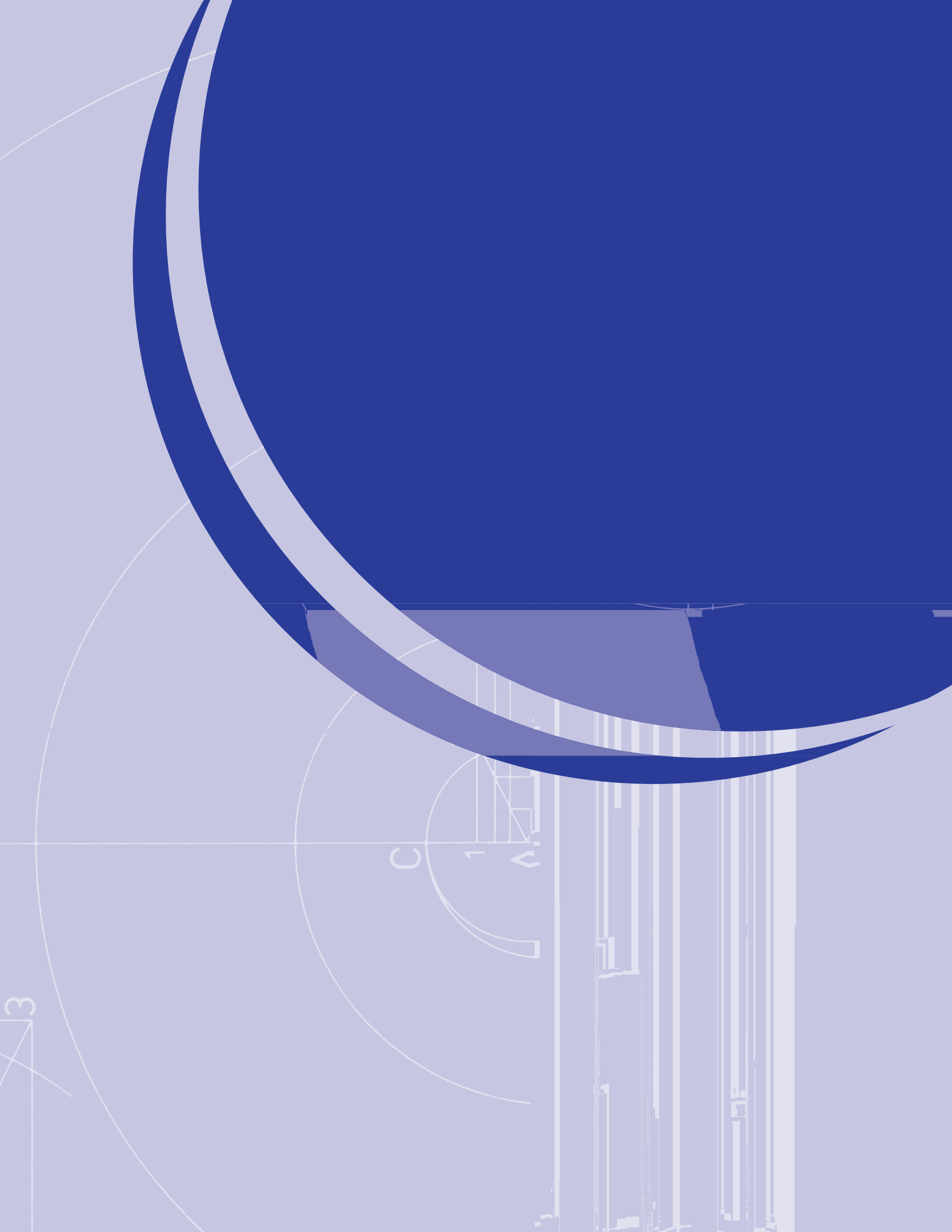

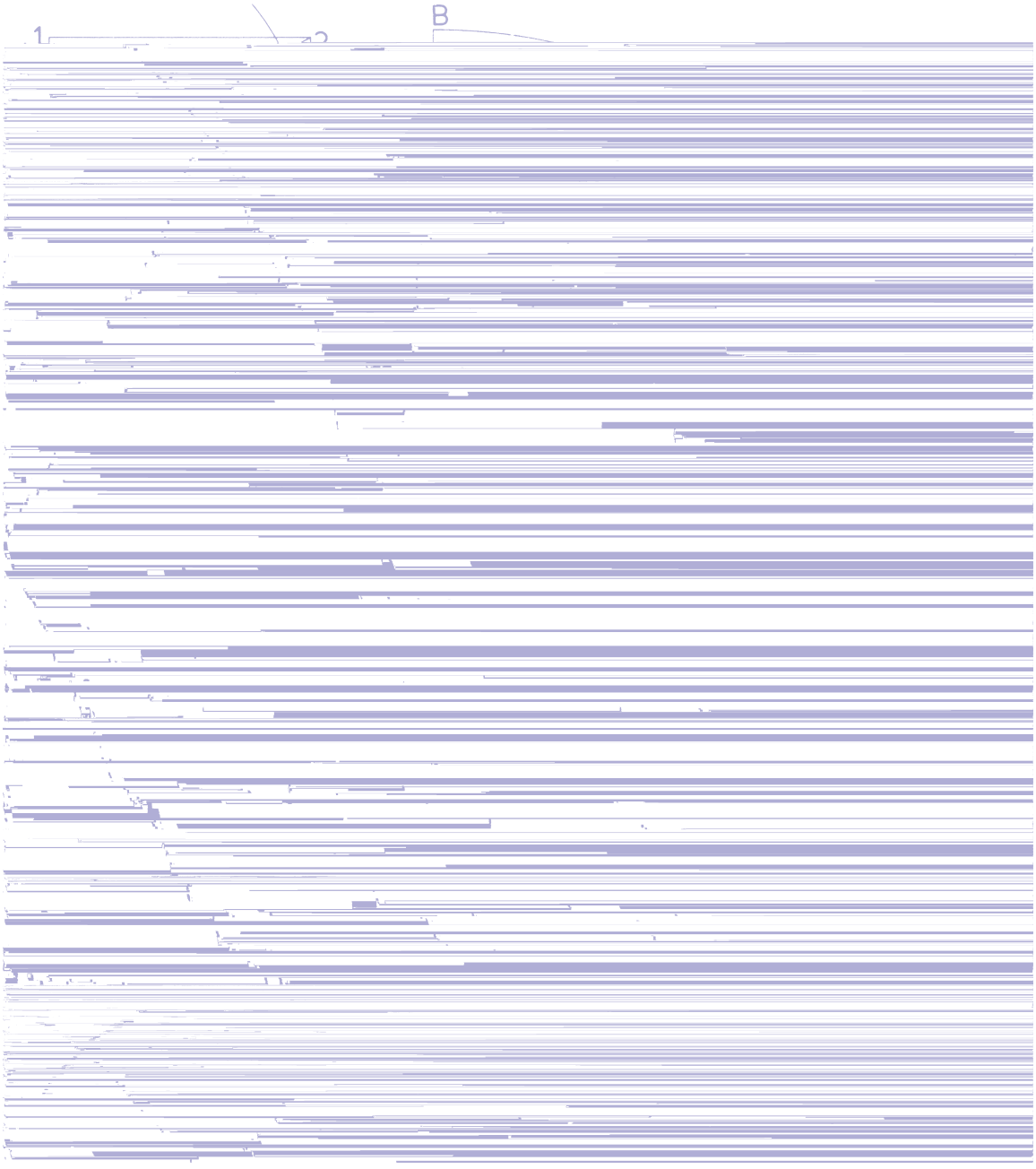

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Overview

The mathematics assessment framework for TIMSS 2003 is framed by two organizing dimensions, a content dimension and a cognitive dimension, analogous to those used in the earlier TIMSS assessments.[1] As outlined below, each dimension has several domains:

Mathematics Content Domains


Number


Algebra


Measurement


Geometry


Data

Mathematics Cognitive Domains


Knowing Facts and Procedures


Using Concepts


Solving Routine Problems


Reasoning

The two dimensions and their domains are the foundation of the mathematics assessment. The content domains define the specific mathematics subject matter covered by the assessment, and the cognitive domains define the sets of behaviors expected of students as they engage with the mathematics content. Each content domain has several topic areas (i.e., number is further categorized by whole numbers, fractions and decimals, integers, and ratio, proportion, and percent). Each topic area is presented as a list of objectives covered in a majority of participating countries, at either grade 4 or grade 8.[2]

Exhibit 2 shows the target percentages of testing time devoted to each content and cognitive domain for both the fourth and eighth grade assessments. The content and cognitive domains for the mathematics assessment are discussed in detail in the following sections. Example mathematics items and tasks are presented in Appendix B.

1 Similarly, the curriculum frameworks for TIMSS 1995 and 1999 assessments included content areas and performance expectations (Robitaille, D.F., et al, (1993), *TIMSS Monograph No.1: Curriculum Frameworks for Mathematics and Science*, Vancouver, BC: Pacific Educational Press).

2 More information about the factors considered in finalizing the topics and assessment objectives is provided in the Introduction.

Number

Grade 4

- Represent whole numbers using words, diagrams, or symbols, including recognizing and writing numbers in expanded form.
- Demonstrate knowledge of place value.
- Compare and order whole numbers.
- Identify sets of numbers according to common properties such as odd and even, multiples, or factors.
- Compute with whole numbers.
- Estimate computations by approximating the numbers involved.
- Solve routine and non-routine problems, including real-life problems.

Grade 8

- Demonstrate knowledge of place value and of the four operations.
- Find and use factors or multiples of numbers, and identify prime numbers.
- Express in general terms and use the principles of commutativity, associativity, and distributivity.
- Evaluate powers of numbers, and square roots of perfect squares to 144.
- Solve problems by computing, estimating, or approximating.

Number: Fractions and Decimals

Grade 4

- Recognize fractions as parts of unit wholes, parts of a collection, locations on number lines, divisions of whole numbers.
- Identify equivalent fractions.
- Compare and order fractions.
- Show understanding of decimals.
- Represent fractions or decimals using words, numbers, or models.
- Add and subtract fractions with the same denominator.
- Add and subtract with decimals.

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Grade 8

- Compare and order fractions.
- Compare and order decimals.
- Demonstrate knowledge of place value for decimals.
- Represent decimals and fractions using words, numbers, or models (including number lines).
- Recognize and write equivalent fractions.
- Convert fractions to decimals and vice versa.
- Relate operations with fractions or decimals to situations and models.
- Compute with fractions and decimals, including use of commutativity, associativity, and distributivity.
- Approximate decimals to estimate computations.
- Solve problems involving fractions.
- Solve problems involving decimals.

Number: Integers

Grade 4

- Not assessed.

Grade 8

- Represent integers using words, numbers, or models (including number lines).
- Compare and order integers.
- Show understanding of addition, subtraction, multiplication, and division with integers.
- Compute with integers.
- Solve problems using integers.

Number: Ratio, Proportion, and Percent

Grade 4

- Solve problems involving simple proportional reasoning.

Grade 8

- Identify and find equivalent ratios.
- Divide a quantity in a given ratio.
- Convert percents to fractions or decimals, and vice versa.
- Solve problems involving percents.
- Solve problems involving proportions.

Algebra

While functional relationships and their uses for modeling and problem solving are of prime interest, it is also important to assess how well the supporting knowledge and skills have been learned. The algebra content domain includes patterns and relationships among quantities, using algebraic symbols to represent mathematical situations, and developing fluency in producing equivalent expressions and solving linear equations.

Because algebra is generally not taught as a formal subject in primary school, this content domain will be identified as *Pa ˙ , E, a , a d R˙a h,* at the fourth grade. In contrast, at the eighth grade the algebra reporting category will reflect understandings across all of the topic areas below.

The major topic areas in algebra are:

- Patterns
- Algebraic expressions
- Equations and formulas
- Relationships

Students will be asked to recognize and extend patterns and relationships. They will also be asked to recognize and use symbols to represent situations algebraically. At the fourth grade, understandings related to patterns, simple equations, and the idea of functions as they apply to pairs of numbers are included. Algebraic concepts are more formalized at the eighth grade, and students should focus on understanding linear relationships and the concept of variable. Students at this level are expected to use and simplify algebraic formulas, solve linear equations and inequalities and pairs of simultaneous equations involving two variables, and use a range of linear and nonlinear functions. They should be able to solve real-world problems using algebraic models and to explain relationships involving algebraic concepts.

Grade 4

- Extend and find missing terms of numeric and geometric patterns.
- Match numeric and geometric patterns with descriptions.
- Describe relationships between adjacent terms in a sequence or between the number of the term and the term.
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Algebra: Algebraic Expressions

Grade 4

- Not assessed.

Grade 8

- Find sums, products, and powers of expressions containing variables.
- Evaluate expressions for given numeric values of the variable(s).
- Simplify or compare algebraic expressions to determine equivalence.
- Model situations using expressions.

Grade 4

- Show understanding of equality using equations, areas, volumes, masses/weights.
- Find the missing number in an equation (e.g., if 17 + __ = 29, what number would go in the blank to make the equation true?).
- Model simple situations involving unknowns with

Measurement

Measurement involves assigning a numerical value to an attribute of an object. The focus of this content domain is on understanding measurable attributes and demonstrating familiarity with the units and processes used in measuring various attributes. Measurement is important to many aspects of everyday life.

The measurement content domain is comprised of the following two main topic areas:

- Attributes and units
- Tools, techniques, and formulas

A measurable attribute is a characteristic of an object that can be quantified. For example, line segments have length, plane regions have area, and physical objects have mass. Learning about measurement begins with a realization of the need for comparison and the fact that different units are needed to measure different attributes. The types of units that students use for measuring and the ways they use them should expand and shift as students move through the curriculum.

At both the fourth and eighth grades, age-appropriate performances expected of students include the use of instruments and tools to measure physical attributes, including length, area, volume, weight/mass, angle, temperature, and time, in standard or non-standard units and converting between systems of units. Students at the fourth grade are expected to use approximation and estimation, and simple formulas, to calculate areas and perimeters of squares and rectangles. At the eighth grade, the measurement domain is expanded to include the measurement of rate (such as speed or density) as well as the application of more complex formulas to measure compound areas and the surface areas of solids.

Measurement: Attributes and Units

Grade 4

- Use given non-standard units to measure length, area, volume, and time (e.g., paper clips for length, tiles for area, sugar cubes for volume).
- Select appropriate standard units to measure length, area, mass/weight,* angle, and time (e.g., kilometers for car trips, centimeters for human height).
- Use conversion factors between standard units (e.g., hours to minutes, grams to kilograms).
- Recognize that total measures of length, area, volume, angle, and time do not change with position, decomposition into parts, or division.

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Grade 8

- Select and use appropriate standard units to find measures of length, area, volume, perimeter, circumference, time, speed, density, angle, mass/weight.*
- Use relationships among units for conversions within systems of units, and for rates.

Measurement: Tools, Techniques, and Formulas

Grade 4

- Use instruments with linear or circular scales to measure length, weight, time, and temperature in problem situations (e.g., dimensions of a window, weight of a parcel).
- Estimate length, area, volume, weight, and time in problem situations (e.g., height of a building, volume of a block of material).
- Calculate areas and perimeters of squares and rectangles of given dimensions.
- Compute measurements in simple problem situations (e.g., elapsed time, change in temperature, difference in height or weight).

Grade 8

- Use standard tools to measure length, weight, time, speed, angle, and temperature in problem situations and to draw line segments, angles, and circles of given size.
- Estimate length, circumference, area, volume, weight, time, angle, and speed in problem situations (e.g., circumference of a wheel, speed of a runner).
- Compute with measurements in problem situations (e.g., add measures, find average speed on a trip, find population density).
- Select and use appropriate measurement formulas for perimeter of a rectangle, circumference of a circle, areas of plane figures (including circles), surface area and volume of rectangular solids, and rates.
- Find measures of irregular or compound areas by covering with grids or dissecting and rearranging pieces.
- Give and interpret information about precision of measurements (e.g., upper and lower bounds of a length reported as 8 centimeters to the nearest centimeter).

Geometry

Even at the fourth grade, the geometry content domain extends well beyond identification of geometric shapes. At both the fourth and eighth

Geometry: Two- and Three-dimensional Shapes

Grade 4

- Know and use vocabulary associated with familiar two- and three-dimensional shapes.
- Identify common geometric shapes in the environment.
- Classify two- and three-dimensional shapes according to their properties.
- Know properties of geometric figures and use them to solve routine problems.
- Decompose shapes and rearrange the parts to form simpler shapes.

Grade 8

- Recall properties of geometric shapes: triangles (scalene, isosceles, equilateral, right-angled) and quadrilaterals (scalene, trapezoid, parallelogram, rectangle, rhombus, square).
- Use properties of familiar geometric shapes in a compound figure to make conjectures about properties of the compound figure.
- Recall properties of other polygons (regular pentagon, hexagon, octagon, decagon).
- Construct or draw triangles and rectangles of given dimensions.
- Apply geometric properties to solve routine and non-routine problems.
- Use Pythagorean theorem (not proof) to solve problems (e.g., find the length of a side of a right-angled triangle given the lengths of the other two sides; or, given the lengths of three sides of a triangle, determine whether the triangle is right-angled).

Geometry: Congruence and Similarity

Grade 4

- Identify triangles that have the same size and shape (congruent).
- Identify triangles that have the same shape but different sizes (similar).

Grade 8

- Identify congruent triangles and their corresponding measures.
- Identify congruent quadrilaterals and their corresponding measures.
- Consider the conditions of congruence to determine whether triangles with given corresponding measures (at least three) are congruent.
- Identify similar triangles and recall their properties.
- Use properties of congruence in mathematical and practical problem situations.
- Use properties of similarity in mathematical and practical problem situations.

Data: Data Representation

Grade 4

- Read data directly from tables, pictographs, bar graphs, and pie charts.
- Display data using tables, pictographs, and bar graphs.
- Compare and match different representations of the same data.

Grade 8

- Read data from charts, tables, pictographs, bar graphs, pie charts, and line graphs.
- Display data using charts, tables, pictographs, bar graphs, pie charts, and line graphs.
- Compare and match different representations of the same data.

Data: Data Interpretation

Grade 4

- Compare characteristics of related data sets (e.g., given data or representations of data on student heights in two classes, identify the class with the shortest/tallest person).
- Draw conclusions from data displays.

Grade 8

- Compare characteristics of data sets, using mean, median, range, and shape of distribution (in general terms).
- Interpret data sets (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points).
- Evaluate interpretations of data with respect to correctness and completeness of interpretation.
- Use and interpret data sets to answer questions.

Data: Uncertainty and Probability

Grade 4

- Not assessed.

Grade 8

- Judge the likelihood of an event as certain, more likely, equally likely, less likely, or impossible.
- Use data from experiments to estimate probabilities for favorable outcomes.
- Use problem conditions to calculate theoretical probabilities for possible outcomes.

Mathematics Cognitive Domains

To respond correctly to TIMSS test items, students will need to be familiar with the mathematics content of the items. Just as important, items will be designed to elicit the use of particular cognitive skills. Many of these skills and abilities are included with topics in the lists of assessable topics that comprise the content domains. However, as an aid in developing balanced tests in which appropriate weight is given to each cognitive domain across all topics, a full set of the learning outcomes mathematics educators would wish to see demonstrated is indispensable. Thus, descriptions of skills and abilities that make up the cognitive domains and will be assessed in conjunction with content are presented in the framework in some detail. These skills and abilities should play a central role in developing items and achieving a balance across the item sets at grades 4 and 8.

The student behaviors used to define the mathematics frameworks have been classified into the following four cognitive domains that are described in this section:

- Knowing Facts and Procedures
- Using Concepts
- Solving Routine Problems
- Reasoning

The specific student behaviors included in each cognitive domain comprise the outcomes sought by educational planners and practitioners around the world. Different groups within a society, and even among mathematics educators, have different views about the relative values of the cognitive skills, or at least about the relative emphases that should be accorded them in schools. For TIMSS they are all regarded as important, and a variety of test items will be used to measure each of them.

The skills and abilities included in each cognitive domain exemplify those that students could be expected to demonstrate in the TIMSS achievement tests. They are intended to apply to both grades 4 and 8, even though the accepted degree of sophistication in demonstrating behaviors will vary considerably between the two. The

Using Concepts

Familiarity with mathematical *c* *c*[illegible] is essential for the effective use of mathematics for problem solving, for reasoning, and thus for developing mathematical understanding.

Knowledge of concepts enables students to make connections between elements of knowledge that, at best, would otherwise be retained as isolated facts. It allows them to make extensions beyond their existing knowledge, judge the validity of mathematical statements and methods, and create mathematical representations. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations is fundamental to success in the subject.

Know	Know that length, area, and volume are conserved under certain conditions; have an appreciation of concepts such as inclusion and exclusion, generality, equal likelihood, representation, proof, cardinality and ordinality, mathematical relationships, place value.
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Classify	Classify/group objects, shapes, numbers, expressions, and ideas according to common properties; make correct decisions about class membership; order numbers and objects by attributes.
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Represent	Represent numbers using models; display mathematical information or data in diagrams, tables, charts, graphs; generate equivalent representations for a given mathematical entity or relationship.
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Solving Routine Problems

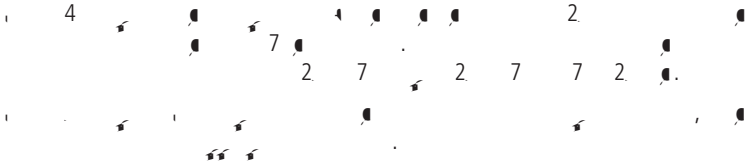
Students should be educated to recognize mathematics as an immense achievement of humanity, and to appreciate its nature. Nevertheless, mathematical knowledge for its own sake is probably not the most compelling reason for universal inclusion of mathematics in school curricula. Prime reasons for inclusion of mathematics are the increasing awareness that effectiveness as a citizen and success in the workplace are greatly enhanced by knowing and, more important, being able to use mathematics. The number of vocations that demand a high level of proficiency in the use of mathematics, or mathematical modes of thinking, has burgeoned with the advance of technology, and with modern management methods.

Problem solving is a central aim, and often means, of teaching school mathematics, and hence this and supporting skills (e.g., manipulate expressions, select, model, verify/check) feature prominently in the [illegible] domain. In items aligned with this domain, the problem settings are more routine than those aligned with the reasoning domain. The routine problems will have been standard in classroom exercises designed to provide practice in particular methods or techniques. Some of these problems will have been in words that set the problem situation in a quasi-real context. Solution of other such "textbook" type problems will involve extended knowledge of mathematical properties (e.g., solving equations). Though they range in difficulty, each of these types of "textbook" problems is expected to be sufficiently familiar to students that they will essentially involve selecting and applying learned procedures.

Problem solving is a desired outcome of mathematics instruction linked with many mathematics topics in the TIMSS framework. Problems may be set in real-life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets. Therefore, problem solving has been included not only in solving routine problems but also in reasoning, depending on whether students are asked to solve routine problems or more non-routine problems (see following).

Solving Routine Problems

Select Select/use an efficient method or strategy for solving problems where there is a known algorithm or method of solution, i.e., an algorithm or method students at the target level could be expected to be familiar with. Select appropriate algorithms, formulas, or units.



Model	Generate an appropriate model, such as an equation or diagram, for solving a routine problem.
Interpret	Interpret given mathematical models (equations, diagrams, etc.); follow and execute a set of mathematical instructions.

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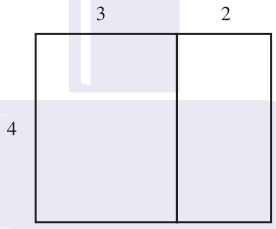

Apply	Apply knowledge of facts, procedures, and concepts to solve routine mathematical (including real-life) problems, i.e., problems similar to those target students are likely to have encountered in class.
Verify/Check	Verify/check the correctness of the solution to a problem; evaluate the reasonableness of the solution to a problem.

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Reasoning

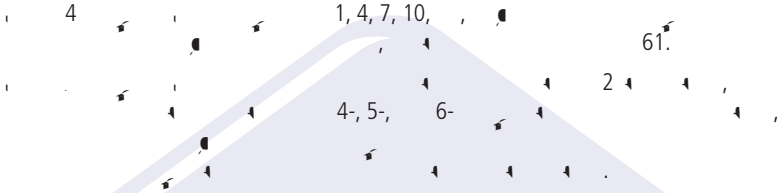
Reasoning mathematically involves the capacity for logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to non-routine problems. Non-routine problems are problems that are very likely to be unfamiliar to students. They make cognitive demands over and above those needed for solution of routine problems, even when the knowledge and skills required for their solution have been learned. Non-routine problems may be purely mathematical or may have real-life settings. Both types of items involve transfer of knowledge and skills to new situations, and interactions among reasoning skills are usually a feature.

Most of the other behaviors listed within the reasoning domain are those that may be drawn on in thinking about and solving such problems, but each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules, and justifying results.

Reasoning

Hypothesize/Conjecture/ Predict	Make suitable conjectures while investigating patterns, discussing ideas, proposing models, examining data sets; specify an outcome (number, pattern, amount, transformation, etc.) that will result from some operation or experiment before it is performed.
Analyze	Determine and describe or use relationships between variables or objects in mathematical situations; analyze univariate statistical data; decompose geometric figures to simplify solving a problem; draw the net of a given unfamiliar solid; make valid inferences from given information.
Evaluate	Discuss and critically evaluate a mathematical idea, conjecture, problem solving strategy, method, proof, etc.

Generalize	Extend the domain to which the result of mathematical thinking and problem solving is applicable by restating results in more general and more widely applicable terms.



Communicating Mathematically

Communicating mathematical ideas and processes is another set of skills that is seen as important for many aspects of living and fundamental to the teaching and learning of the subject. Representing, modeling, and interpreting, for example, are aspects of communication. While communication is an important outcome of mathematics education, it is not included as a separate cognitive domain. Rather, it may be thought of as an overarching dimension across all mathematics content areas and processes. Communication is fundamental to each of the other categories of *ac a d cd* , *c c* , *b* , and *a* - , and students' communication in and about mathematics should be regarded as assessable in each of these areas.

Students in TIMSS may demonstrate communication skills through description and explanation, such as describing or discussing a mathematical object, concept, or model. Communication also occurs in using mathematical terminology and notation; demonstrating the procedure used in simplifying, evaluating, or solving an equation; or using particular representational modes to present mathematical ideas. Communication is involved in explaining why a particular procedure or model has been used, and the reason for an unexpected or anomalous result. While describing and explaining are not explicitly listed as behaviors in the framework document, items across a wide range of content and processes will require these communication skills. Students could be asked to describe or explain why they chose a particular method, why they made a particular response, and so on.

Guidelines for Calculator Use

Although technology in the form of calculators and computers can help students learn mathematics, it should not be used to replace basic understanding and competencies. Like any teaching tool, calculators need to be used appropriately, and policies for their use differ across the TIMSS countries. Also, the availability of calculators varies widely. It would not be equitable to require calculator use when students in some countries may never have used them. Similarly, however, it is not equitable to deprive students of the use of a familiar tool. Thus, after considerable debate on this issue, beginning with TIMSS 2003 calculators are permitted but not required for newly developed grade 8 assessment materials.

The aim of the TIMSS guidelines for calculator use is to give students the best opportunity to operate in settings that mirror their classroom experience. Thus, if students are used to having calculators for their classroom activities, then the country should encourage students to use them during the assessment. On the other hand, if students are not used to having calculators or are not permitted to use them in their daily mathematics lessons, then the country need not permit their use. In developing the new assessment materials, every effort will be made to ensure that the test questions do not advantage or disadvantage students either way – with or without calculators.

Students at grade 4 will not be permitted to use calculators. Since calculators were not permitted at grade 8 in the 1995 and 1999 assessments, test administration procedures will ensure that they are not available for items from those assessments used to measure trends.